Connect and Reflect
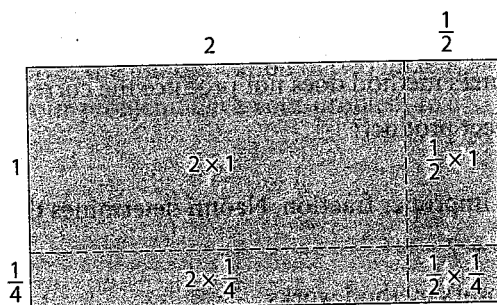
Key Ideas

- You can model the multiplication of two mixed numbers or improper fractions using partial areas of a rectangle.

$$2\frac{1}{2} \times 1\frac{1}{4} = 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$$
$$= 3\frac{1}{8}$$

- You can estimate the product of two mixed numbers or improper fractions by multiplying the whole numbers closest to them.

$$3\frac{1}{4} \times 1\frac{3}{4} \approx 3 \times 2$$
$$\approx 6$$

- A rule for multiplying two mixed numbers is to express them as improper fractions and then multiply the numerators and multiply the denominators.

$$1\frac{2}{3} \times 2\frac{1}{5} = \frac{5}{3} \times \frac{11}{5}$$
$$= \frac{55}{15}$$
$$= \frac{11}{3}$$

Practise

For help with #1 and #2, refer to Example 1 on pages 151–152.

1. Use a diagram to determine each product.

a) $1\frac{1}{3} \times \frac{3}{4}$ b) $2\frac{1}{2} \times 1\frac{3}{5}$

c) $1\frac{1}{3} \times 1\frac{1}{2}$ d) $2\frac{1}{2} \times 2\frac{1}{4}$

2. Determine each product using a diagram.

a) $\frac{1}{2} \times 2\frac{1}{2}$

b) $2\frac{1}{3} \times 2\frac{1}{3}$

c) $1\frac{1}{2} \times 2\frac{1}{3}$

d) $1\frac{1}{5} \times 1\frac{1}{2}$

For help with #3 and #4, refer to Example 2 on page 152.

3. Calculate each product. Then check if your answer is reasonable.

a) $5 \times 3\frac{3}{4}$ b) $\frac{4}{5} \times \frac{10}{7}$

c) $2\frac{1}{5} \times 1\frac{2}{3}$ d) $\frac{5}{6} \times 2\frac{1}{4}$

4. Calculate each product. Then check if your answer is reasonable.

a) $\frac{8}{3} \times \frac{11}{6}$

b) $2\frac{5}{6} \times 4$

c) $6\frac{1}{2} \times 3\frac{1}{2}$

5. Is it possible to multiply two mixed numbers without converting them to improper fractions or without using a diagram? Explain.

6. Henri multiplies $2\frac{1}{2} \times 3\frac{1}{4}$ as follows: $2 \times 3 = 6$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$, so $2\frac{1}{2} \times 3\frac{1}{4} = 6\frac{1}{8}$.

   a) Explain why Henri's method does not produce the correct product.

   b) What is the correct product?

7. To express $4\frac{2}{3}$ as an improper fraction, Naomi determines the numerator by calculating $3 \times 4 + 2$.

   a) Explain why Naomi's method works.

   b) Use your explanation to write a rule for expressing a mixed number as an improper fraction. Test your rule.

   c) Write a rule for expressing an improper fraction as a mixed number. Test your rule.

8. Two and a half laps of a running track equal 1 km. Write an expression and determine how many laps equal 3 km.

9. Earth turns on its axis once every 24 h. How many hours does Earth take to complete $2\frac{1}{4}$ turns?

10. One summer day in Masset, BC, had $10\frac{1}{2}$ h of daylight. It was sunny for $\frac{1}{3}$ of that time. For how many hours was it sunny that day?

Masset

11. Alexa takes $\frac{1}{4}$ h to ride her bicycle to her friend's house. If Alexa walks instead, the trip takes her $2\frac{1}{2}$ times as long. How long does Alexa take to walk to her friend's house

    a) in hours?

    b) in minutes?

12. ✓ **Competency Check** In Eric's apartment, the living room is 1.75 times as long and 2.5 times as wide as the den. Eric is buying the same type of carpet for both rooms. How many times as much will the carpet cost for the living room as for the den? Model the problem and explain your solution.

13. Argon has 18 protons. Zinc has $1\frac{2}{3}$ times as many protons as argon. Cadmium has $1\frac{3}{5}$ times as many protons as zinc. How many protons do the three types of atoms have altogether?
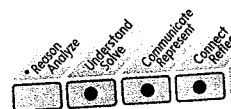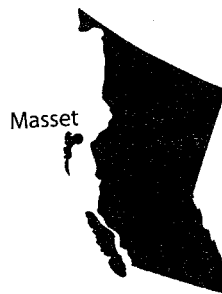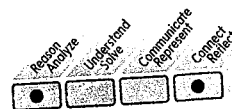
14. A corner store buys goods at the wholesale price and sells them for $\frac{7}{5}$ of the wholesale price. The wholesale price of a case of 12 cans of stew is $15. For how much does the store sell 1 can of stew?

15. If you multiply a mixed number and a proper fraction, how does each value compare with the value of the product?

16. Create your own word problem that involves the multiplication of two mixed numbers. Make sure that you can solve your problem. Give your problem to a classmate to solve.

Extend

17. Moira multiplies $2\frac{1}{3} \times 2\frac{1}{2}$ as follows:

$$2\frac{1}{3} \times 2\frac{1}{2} = \frac{7}{3} \times \frac{5}{2}$$
$$= \frac{14}{6} \times \frac{15}{6}$$
$$= \frac{210}{36}$$
$$= \frac{35}{6}$$
$$= 5\frac{5}{6}$$
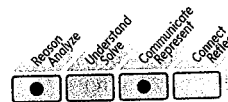
a) How could you use estimation to determine if Moira made a mistake?

b) Is this the most efficient way to do the calculation? Explain.

18. Write an expression to describe each pattern. Determine the next three terms in each pattern.

a) $4\frac{1}{3}, 2\frac{1}{6}, 1\frac{1}{12}, \frac{13}{24}, \dots$ **b)** $4, 6, 9, 13\frac{1}{2}, \dots$

19. a) Describe a situation outside of the mathematics classroom in which you may have to multiply two mixed numbers.

b) Describe a situation outside of the mathematics classroom in which you may have to multiply three mixed numbers.

20. Calculate.

a) $4 \times 1\frac{1}{2} \times 2\frac{1}{2}$ **b)** $\frac{2}{3} \times 3\frac{1}{3} \times 4\frac{1}{2}$

c) $2\frac{3}{4} \times 1\frac{1}{3} \times 3\frac{1}{2}$ **d)** $1\frac{1}{6} \times 1\frac{2}{5} \times 2\frac{2}{7}$

21. Determine a mixed number that makes the equation true.

a) $1\frac{2}{3} \times \blacksquare = 2\frac{1}{2}$ **b)** $\blacksquare \times 2\frac{1}{6} = 2\frac{3}{5}$

c) $\blacksquare \times 1\frac{1}{4} = 3\frac{1}{8}$ **d)** $2\frac{1}{3} \times \blacksquare = 5\frac{5}{6}$