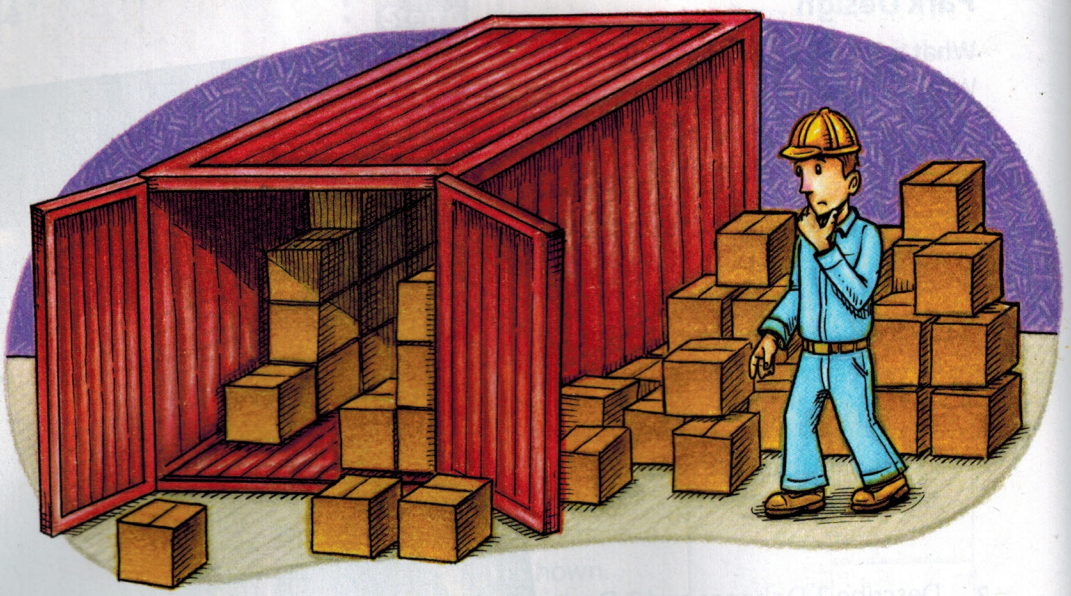
7.1 Understanding Volume



Focus on...

After this lesson, you will be able to...

- explain the meaning of volume
- determine the volume of a right rectangular prism, right triangular prism, and right cylinder
- show that orientation does not affect volume

Bruce has just taken on a part-time job at a local shipping company. He is packing boxes into a shipping container. He knows how many boxes he can fit on the bottom of the container. How can he use this information to figure out how many boxes the shipping container will hold?

Materials

- centimetre cubes

base (of a prism or cylinder)

- any face of a prism that shows the named shape of the prism
- the base of a rectangular prism is any face
- the base of a triangular prism is a triangular face.
- the base of a cylinder is a circular face

height (of a prism or cylinder)

- the perpendicular distance between the two bases of a prism or cylinder

Explore the Math

How does the area of the base of a right prism relate to its volume?

1. a) Use centimetre cubes to build models of four different right rectangular prisms.
   b) What is the area of the **base** for each model? Record your data.
   c) What is the **height** of each model? Record your data.

2. How does the number of cubes help to determine the **volume** of each rectangular prism? What is the volume of each prism? Record your data.

One centimetre cube is equal to 1 cm^3.

volume
- the amount of space an object occupies
- measured in cubic units

Reflect on Your Findings

4. **a)** What is the relationship between the area of the base, the height of the prism, and the volume of a rectangular prism?

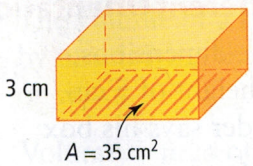
   b) Do you think this same relationship exists for the volume of a right triangular prism? Explain your reasoning.

Literacy Link
Read 1 cm^3 as "one cubic centimetre."

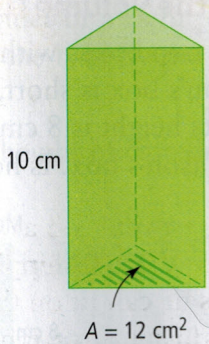
Example 1: Determine the Volume Using the Base and the Height

Determine the volume of each right prism or cylinder.

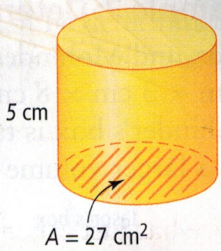
a)



b)



c)



Literacy Link
Prisms and cylinders in this chapter are *right* prisms and *right* cylinders.

Solution

a) The prism is a right rectangular prism.
The area of the rectangular base is 35 cm^2.
The height of the prism is 3 cm.
Volume = area of base × height of prism
$V = 35 \times 3$
$V = 105$
The volume of the right rectangular prism is 105 cm^3.

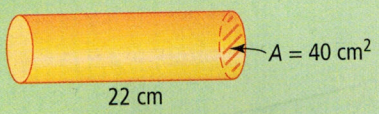
Why are the units for volume in cm^3?

b) The prism is a right triangular prism.
The area of the triangular base is 12 cm^2.
The height of the prism is 10 cm.
Volume = area of base × height of prism
$V = 12 \times 10$
$V = 120$
The volume of the right triangular prism is 120 cm^3.

c) The cylinder is a right cylinder.
The area of the circular base is 27 cm^2.
The height of the cylinder is 5 cm.
Volume = area of base × height of cylinder
$V = 27 \times 5$
$V = 135$
The volume of the right cylinder is 135 cm^3.

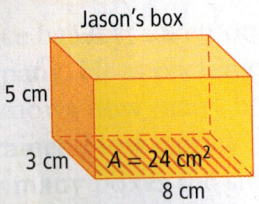
Show You Know

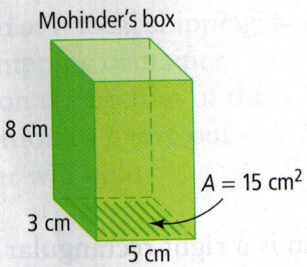
What is the volume of the right cylinder?



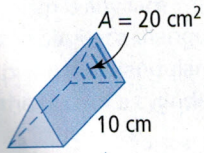
Example 2: Determine the Volume Using Different Orientations

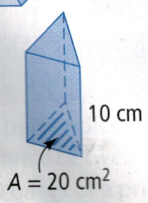
Jason and Mohinder have two boxes with the same dimensions, 5 cm × 3 cm × 8 cm. Jason's box is short, with a height of 5 cm. Mohinder's box is taller; its height is 8 cm. Mohinder says his box has a larger volume than Jason's box. Is he correct?





orientation
- the different position of an object formed by translating, rotating, or reflecting the object





Solution

Determine the volume of each rectangular prism.

Jason's box: Base area of 24 cm^2
Volume = area of base × height
$V = 24 \times 5$
$V = 120$
The volume of the rectangular prism is 120 cm^3.

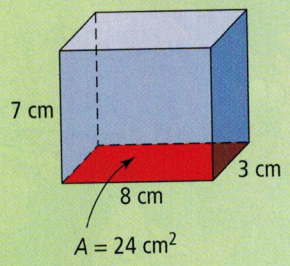
Mohinder's box: Base area of 15 cm^2
Volume = area of base × height
$V = 15 \times 8$
$V = 120$
The volume of the rectangular prism is 120 cm^3.

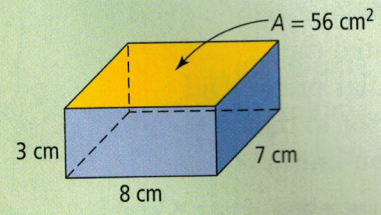
Mohinder is not correct. Both boxes have the same volume.

Do you think changing the **orientation** of a 3-D object ever affects the volume?

Show You Know

Which box has the greater volume? Explain your reasoning.





Key Ideas

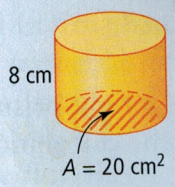
- The volume of a right cylinder or a right prism can be determined by multiplying the area of the base by the height of the cylinder or prism.

Volume = area of base × height of cylinder

$V = 20 \times 8$

$V = 160$

The volume of the cylinder is 160 cm³.

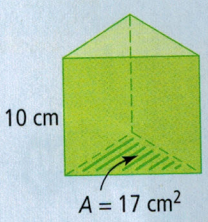


Volume = area of base × height of prism

$V = 17 \times 10$

$V = 170$

The volume of the triangular prism is 170 cm³.



- Changing the orientation of a 3-D object does not affect its volume.

Volume = area of base × height

$V = 54 \times 4$

$V = 216$

The volume of the cylinder is 216 cm³.